محاضرة [8] د. عرفة الرصد 22/11/2015

II] Radix-2 DIT-FFT

This Algorithm is based on 3 properties

1) $W_N^{K+N} = W_N^K$

2) $W_N^{K+N/2} = -W_N^K$

3) $W_N^2 = W_{N/2}$

from the previous Lecture

for $N=4$ $\quad x(n) = \{x(0), x(1), x(2), x(3)\}$

نقسمها لجزء فردي و جزء زوجي

$f_1(n) = \{x(0), x(2)\} = x(2n)$

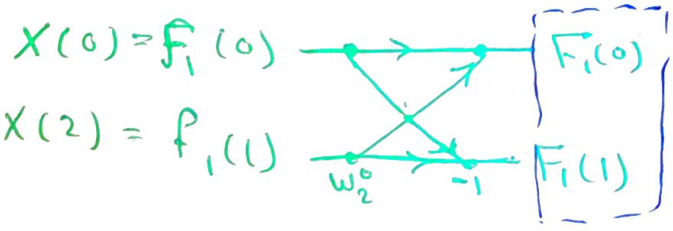
{even numbered sequence}

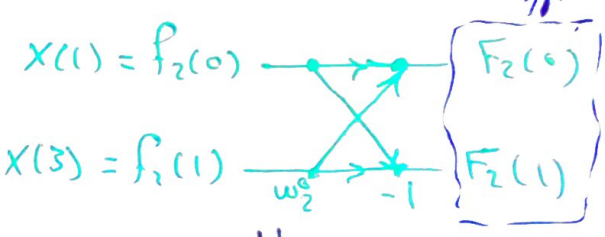
$f_2(n) = \{x(1), x(3)\}$

(odd numbered sequence)

$F_1(K) \Rightarrow$ 2-point DFT

$F_2(K) \Rightarrow$ 2-point DFT





1) $X(K) = F_1(K) + W_N^K F_2(K)$

2) $X(K+\frac{N}{2}) = F_1(K) - W_N^K F_2(K)$

$\Rightarrow K = 0, 1, 2, \ldots$

بالتعويض عن K بـ 0, 1 في كل معادلة من (1) و (2)

(1)

$X(0) = F_1(0) + W_4^0 F_2(0)$

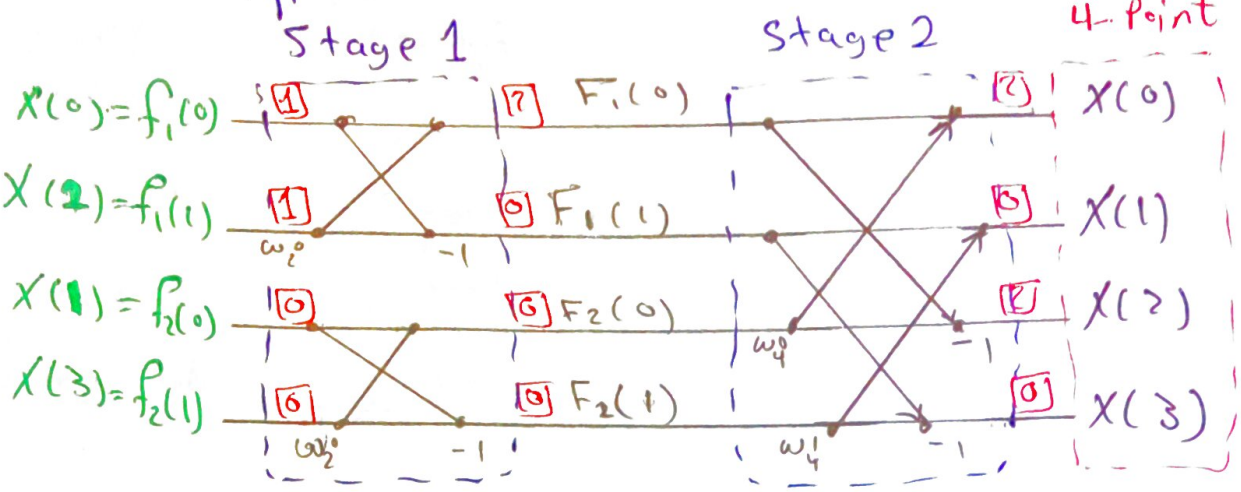
$X(1) = F_1(1) + W_4^1 F_2(1)$

(2)

$X(2) = F_1(0) - W_4^0 F_2(0)$

$X(3) = F_1(1) - W_4^1 F_2(1)$

$F_1(K)$ & $F_2(K)$



$N = 4$ ← Butterfly Comp. عدد العمليات

[The Signal graph with Butterfly computation for $N = 4$ ---]

Ex: $x(n) = \{1, 0, 1, 0\} \Rightarrow x(K) = \{2, 0, 2, 0\}$

This is solved in red on the graph

For $\boxed{N = 8}$ ← ابدأ هنا

$$X(K) = \{X(0), X(1), X(2) \ldots, X(7)\}$$

افصل الزوجي والفردي

$f_1(n) = \{X(0), X(2), X(4), X(6)\}$ — $\frac{N}{2}$ point = 4 point — even numbered sequence

$f_2(n) = \{X(1), X(3), X(5), X(7)\}$

① $X(K) = F_1(K) + W_N^K F_2(K)$

② $X(K + N/2) = F_1(K) - W_N^K F_2(K)$

$\Rightarrow K = 0, 1, 2, 3$ ($\frac{N}{2} - 1$)

بالتعويض بقيم K في كل معادلة ينتج 8 معادلات

for eq ①

$$X(0) = F_1(0) + W_8^0 \; F_2(0)$$
$$X(1) = F_1(1) + W_8^1 \; F_2(1)$$
$$X(2) = F_1(2) + W_8^2 \; F_2(2$$
$$X(3) = F_1(3) + W_8^3 \; F_2(3)$$

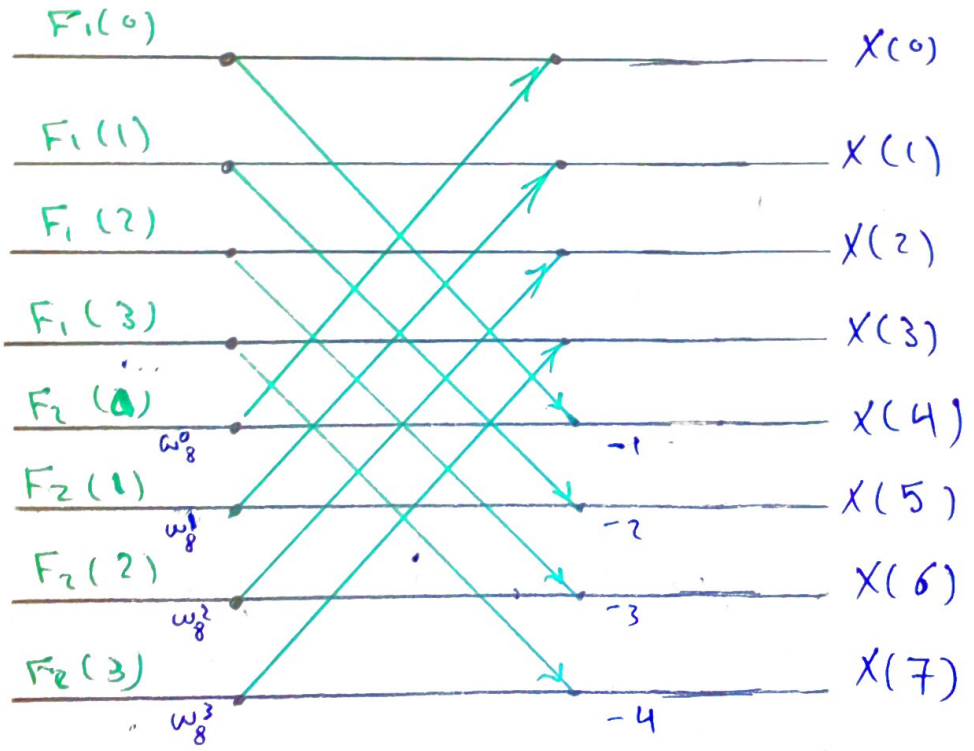
$K = 0, 1, 2, 3$

for eq ②

$K = 0, 1, 2, 3$

$$X(4) = F_1(0) - W_8^0 \; F_2(0)$$
$$X(5) = F_1(1) - W_8^1 \; F(1)$$
$$X(6) = F_1(2) - W_8^2 \; F(2)$$
$$X(7) = F_1(3) - W_8^3 \; F(3)$$

⟸ نكرر نفس اللي فوق بس بإشارة سالبة



⟹ Turn over

$X(n)$

$f_1(n) = \{X(0), X(2), X(4), X(6)\}$ — $f_2(n) = \{X(1), X(3), X(5), X(7)\}$

هجزأ تاني

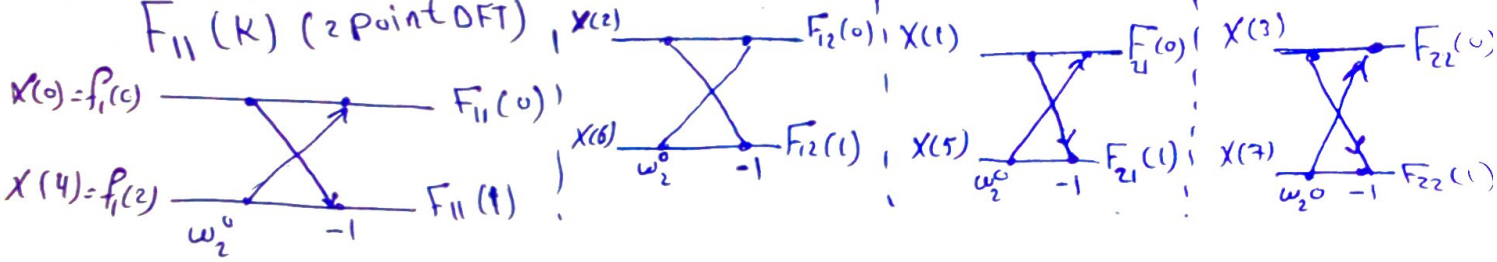
$f_{11}(n) = \{X(0), X(4)\}$ → $f_1(0)$ $f_1(2)$ even numbered

$f_{12}(n) = \{X(2), X(6)\}$ → $f_1(1)$ $f_1(3)$ odd numbered

$f_{21}\{X(1), X(5)\}$ → $f_2(0)$ $f_2(2)$ even numbered

$f_{22} = \{X(3), X(7)\}$ → $f_2(1)$ $f_2(3)$ odd numbered

$F_{11}(K)$ (2 point DFT)



$F_{11}(K)$ $F_{12}(K)$ $F_{21}(K)$ $F_{22}(K)$

$$F_1(K) = F_{11}(K) + W_{N/2}^{K} F_{12}(K) \quad (1)$$

$$F_1(K+N/2) = F_{11}(K) - W_{N/2}^{K} F_{12}(K) \quad (2)$$

$F_1(K)$

$$F_2(K) = F_{21}(K) + W_{N/2}^{K} F_{22}(K)$$

$$F_2(K+N/2) = F_{21}(K) - W_{N/2}^{K} F_{22}(K)$$

$F_2(K)$

$K=0 \Rightarrow F_1(0) = F_{11}(0) + \omega_4^0 F_{12}(0)$ } eq①

$K=1 \Rightarrow F_1(1) = F_{11}(1) + \omega_4^1 F_{12}(1)$ } eq①

$K=2 \Rightarrow F_1(2) = F_{11}(0) - \omega_4^0 F_{12}(0)$ } eq②

$K=3 \Rightarrow F_1(3) = F_{11}(1) - \omega_4^1 F_{12}(1)$ } eq②

$F_2(0) = F_{21}(0) + \omega_4^0 F_{22}(0)$ } eq③

$F_2(1) = F_{21}(1) + \omega_4^1 F_{22}(1)$ } eq③

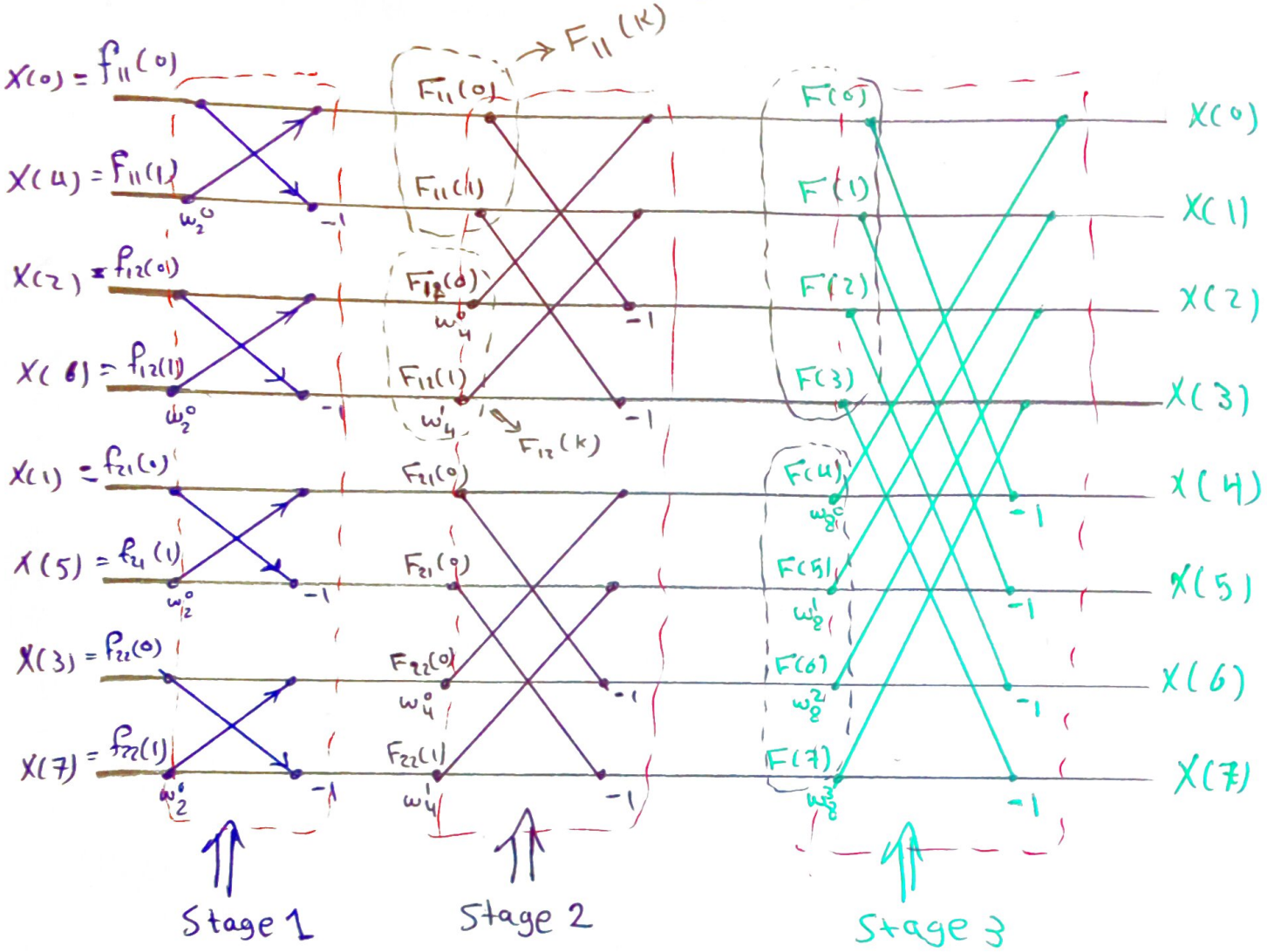
$F_2(2) = F_{21}(0) - \omega_4^0 F_{22}(0)$ } eq④

$F_2(3) = F_{21}(1) - \omega_4^1 F_{22}(1)$ } eq④

⟹ Turn over

Page [3] في Butterfly تكملة الـ

(الخلاصة)



$X(n) = \{X(0), X(1), \ldots, X(7)\}$

لقد رتبت الـ seq. بالطريقة دي ⇐

$f_1 = \{X(0), X(2), X(4), X(6)\}$

$f_2 = \{X(1), X(3), X(5), X(7)\}$

$\{X(0), X(4)\}$ $\{X(2), X(6)\}$ $\{X(1), X(5)\}$ $\{X(3), X(7)\}$

⟹ Turn over

طريقة أخرى لمعرفة الـ seq. العكسي لـ Binary

0	0 0 0		0 0 0	0
1	0 0 1		1 0 0	4
2	0 1 0		0 1 0	2
3	0 1 1	⇒ اكتبها	1 1 0	6
4	1 0 0		0 0 1	1
5	1 0 1		1 0 1	5
6	1 1 0		0 1 1	3
7	1 1 1		1 1 1	7

Notes :-

① * No. of stages = $V = \log_2 N$

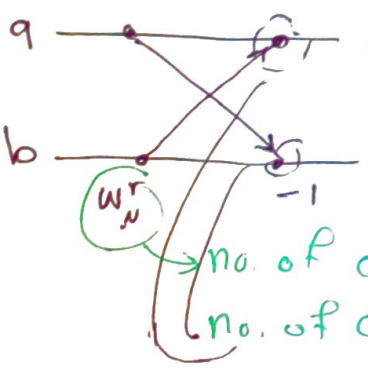
ex ⇒ $N=4 \Rightarrow V = \log_2 4 = 2$

$N=8 \Rightarrow V = \log_2 8 = 3$

② * No. of butterflies for each stage = $\frac{N}{2}$

Total No. of butterflies = $\frac{N}{2} \log_2 N$

③ *



$A = a + W_N^r b$

$B = a - W_N^r b$

no. of complex multiplication = 1

no. of complex Addition = 2

Total no. of complex Multiplication = $\frac{N}{2} \cdot \log_2 N$

Total no. of complex Additions = $2 * \frac{N}{2} \log_2 N$

$= N \log_2 N$

مقارنة

FFT Complex Computation

من الفقرة 3 في الصفحة السابقة
Page 6

* Total no. of complex Multiplication
$= \frac{N}{2} \log_2 N$
* Total no. of Complex Additions
$= N \log_2 N$

Direct DFT

$$X(K) = \sum_{n=0}^{N-1} x(n) W_N^{kn} \quad n = 0, 1, \dots N-1 \quad K = 0, 1, \dots, N-1$$

* Total no. of Complex multiplications
$= N * N$
* Total no. of complex Additions
$= N * (N-1)$

N	Radix-2 DIT FFT	Direct DFT	
4	4	16	no. of complex Mul.
	8	12	no. of Complex Add.
8	12	64	no. of complex Mul.
	24	56	no. of complex Add.

كلما زادت N اتضح الفرق أكثر

16	32	240	mul.
	64	256	Add.
256	1024	65280	mul.
	2048	62536	Add.

$$W_8^0 = 1$$

$$W_8^1 = \frac{1}{2} - j\frac{1}{\sqrt{2}}$$

$$W_8^2 = -j$$

$$W_8^3 = \frac{-1}{\sqrt{2}} - j\frac{1}{\sqrt{2}}$$

$$\left\{ W_N = e^{-j\frac{2\pi}{N}} \right.$$

ex: $x(n) = \{1, 1, 1, 1, 0, 1, 1, 1\}$

$X(K) = \{7, 1, -1, 1, -1, 1, -1, 1\}$